Transition radiation and the origin of sonoluminescence

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It has been shown by Liberati *et al.* [Phys. Rev. D **61**, 85023 (2000)] that a dielectric medium with a time-dependent refractive index may produce photons. We point out that a free electric charge that interacts with such a medium will emit quantum-mechanically modified transition radiation in which an arbitrary odd number of photons will be present. Excited atomic electrons will also exhibit a similarly modified emission spectrum. This effect may be directly observable in connection with sonoluminescence.

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There exist mainly two ''schools'' in the effort to understand and explain sonoluminescence. One school argues in favor of the view that sonoluminescence can be understood on the basis of gas dynamics (see [1], e.g.). In the second main approach it is argued that the flash of light which is emitted from sonoluminescent bubbles is essentially due to energy which is released from the quantum vacuum (see [2], e.g.). In this paper we will to some extent attempt to bridge the gap between these apparently incompatible views.

Our starting point is the physical scenario for explaining sonoluminescence which was presented in [3]. There the authors showed that sufficient energy in principle may be released from the electromagnetic vacuum in a dielectric medium with a time-dependent refractive index so as to account for the radiation emitted from sonoluminescent bubbles. The authors adopted a model in which there is a *homogeneous* medium of time-varying refractive index. A detailed interaction mechanism, whereby this radiation can be released, was not specified. The approach taken in [3] was thus purely phenomenological. We note that there are actually two kinds of phenomenology involved here: (i) already the use of the refractive index as such means that the complicated interaction between field and matter is described in terms of one single scalar parameter, the refractive index. It is this fact that gives rise to peculiar properties of phenomenological electromagnetic theory, such as the spacelike nature of the total four-momentum of a radiation field within a medium (cf., for instance, [4]). We shall assume, such as in [3], that the use of the refractive index is meaningful in a quantum context also (otherwise, we would have to resort to some kind of many-body theory, which would be an extremely complicated approach). From a physical viewpoint one may interpret the refractive index as a time-dependent external field, producing pairs of photons. (ii) The second kind of phenomenology in [3] is the assumption about homogeneity of the medium. Of course, this is a very rough model, but we agree with the authors in that it ought to be a reasonable first step towards a realistic theory. (In a later paper [5], the same authors take finite volume effects into account.)

Characteristic for the above-mentioned approach is that the produced electromagnetic energy has to be calculated via the Bogolubov transformation [6]. The calculational method is essentially the same as that found elsewhere when ''sudden'' changes are involved, as for instance in case of emitted radiation energy from the sudden production of a cosmic string [7].

Our main purpose with the present paper is to make a first step away from the phenomenological level and present a simple account of the photon production in terms of the excited or ionized atoms in the sonoluminescent bubble. This kind of approach may reveal properties of the emission spectrum that are connected with the interaction between vacuum and a time-dependent refractive index. That is, we may obtain some clues telling us about the relationship between the ''hydrodynamic'' and the ''vacuum'' interpretations of sonoluminescence. The present paper is to our knowledge the first attempt to bridge these two interpretations (although in this context reference ought to be made to the very recent paper of Motyka and Sadzikowski [8] dealing with atomic collisions and sonoluminescence).

In this paper we will show that a free electron, which interacts with a dielectric medium with a time-dependent refractive index, induces production of any odd number of photons from the vacuum at the tree level. The process can be seen as a quantum-mechanically (and nonperturbatively) modified classical transition radiation process.[1] Excited atomic electrons will also exhibit a similarly modified emission spectrum. We suggest that these processes may provide partial mechanisms for the observed emission of (at least some of the) photons from sonoluminescent bubbles.

This paper is organized as follows. We will first briefly consider the model that was presented in [3] with a special eye towards the question of a consistent quantization of the electromagnetic field in a medium with a time-dependent

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[1]Two particularly useful references on classical aspects of transition radiation can be found in [9,10].

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refractive index. We then discuss qualitatively the emission of photons from free and bound electrons that live in such a medium. At the end of this paper we point out one experimental consequence of our findings. We also point out a new direction for further theoretical investigations into the question of the possible importance of transition radiation in connection with sonoluminescence.

We follow [3] and choose to write Maxwell's equations in a dielectric medium with a time- and position-independent dielectric constant ϵ, and with the magnetic permeability μ set to unity, as (we set the speed of light, ϵ and μ in the true vacuum to unity in the following)

$$\mathrm{div}\,\mathbf{A}+\epsilon\partial_t\phi=0, \tag{1}$$

$$-\epsilon\partial_t^2\phi+\mathrm{div}(\mathrm{grad}\,\phi)=0, \tag{2}$$

$$-\epsilon\partial_t^2\mathbf{A}+\mathrm{grad}(\mathrm{div}\,\mathbf{A})=\mathbf{0}. \tag{3}$$

ϕ is the electromagnetic scalar potential, and $\mathbf{A}$ is the three-vector potential. The first of these equations can be seen as a ''generalized'' Lorentz condition. We will further *choose* potentials on the classical level such that

$$\phi=0, \quad \mathrm{div}\,\mathbf{A}=0. \tag{4}$$

The resulting wave equation for the electromagnetic three-vector field potential then has solutions in the form

$$\mathbf{A}_{\mathbf{k}\lambda}=N_{\mathbf{k}}\mathbf{e}_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}, \quad \mathbf{k}\cdot\mathbf{A}_{\mathbf{k}\lambda}=0, \quad \lambda\in\{0,1,2,3\}, \tag{5}$$

where $\mathbf{e}_{\mathbf{k}\lambda}$ is the polarization vector, $N_{\mathbf{k}}$ is a normalization constant with respect to some suitably defined inner product, and ω and $\mathbf{k}$ are related by

$$-\epsilon\omega^2+\mathbf{k}^2=0. \tag{6}$$

The polarization vectors can always be chosen such that $\eta_{\alpha\beta}e^{\alpha}{}_{\mathbf{k}\lambda}e^{\beta}{}_{\mathbf{k}\lambda'}=\eta_{\lambda\lambda'}$, where η denotes the Minkowski metric with $\mathrm{sgn}(\eta)=(-1,+1,+1,+1)$. We will let $\lambda=1,2$ refer to the transverse components of the electromagnetic field, while $\lambda=0$ and $\lambda=3$ refer to the scalar and the longitudinal components, respectively.

It was assumed in [3] that the dielectric constant of the medium suddenly changes from its initial constant value, and to another constant final value, i.e., it was assumed that

$$\epsilon(t)\rightarrow\epsilon_{\mathrm{initial}}=\mathrm{const},\ t\rightarrow-\infty \tag{7}$$

$$\epsilon(t)\rightarrow\epsilon_{\mathrm{final}}=\mathrm{const},\ t\rightarrow+\infty. \tag{8}$$

It was shown that such a change in the refractive properties of a dielectric medium may give rise to production of photons. By making the change in ϵ steep enough, it was shown that sufficient energy in principle can be released so as to account for the energy emitted in actual experiments with sonoluminescent bubbles. The model does also reproduce the observed spectrum of the photons which are emitted from such bubbles with a high degree of accuracy.

There is at present a debate in the literature as to whether the *static* Casimir energy is large enough to account for the energy emitted by the sonoluminescing bubble. It was argued in [11] that the static Casimir energy is by far too small. It is only fair to mention, however, that in a series of papers [12] it has been argued that the static Casimir energy is large. As static considerations are not likely to be closely connected with the dynamic sonoluminescence problem, we will not pursue this subject further here.

We will now confront the question of whether the electromagnetic field can be quantized in a consistent fashion when one assumes Eqs. (7) and (8) to hold true. Our main finding is that if one treats the ''generalized'' Lorentz-gauge condition along the same lines as in the standard Gupta-Bleuer formalism, and along with the assumption of a time-dependent dielectric constant in Eqs. (7) and (8), one gets the conditions

$$\phi|\psi\rangle=0, \quad \mathbf{A}_{\parallel}|\psi\rangle=\mathbf{0}, \tag{9}$$

by necessity at $t\rightarrow+\infty$, where $\mathbf{A}_{\parallel}$ is the longitudinal part of the field potential ($|\psi\rangle$ denotes any photon state) *without the need to invoke gauge invariance*, in addition to the condition

$$\mathrm{div}\,\mathbf{A}_{\perp}|\psi\rangle=0, \tag{10}$$

where $\mathbf{A}_{\perp}$ is the transversal part of the field potential. Let us see how this comes about.

In the quasistatic asymptotic regions ($t\rightarrow\pm\infty$) the electromagnetic field can be quantized along the usual lines. This implies the existence of two in general inequivalent descriptions of the quantized field. In the in region at infinite past the field can be quantized as

$$A^{\mu\,\mathrm{in}}=\sum_{j\lambda}\ (a^{\mathrm{in}}_{j\lambda}A^{\mu\,\mathrm{in}}_{j\lambda}+a^{\mathrm{in}\dagger}_{j\lambda}A^{\mu\,\mathrm{in}*}_{j\lambda}), \quad \mu\in\{0,1,2,3\}, \tag{11}$$

where it is assumed a finite quantization volume. Note that this discussion is a general one in which we do *not* impose the constraints in Eq. (4). The $\{A^{\mu\,\mathrm{in}}_{j\lambda}\}$ modes represent a complete set of solutions of the wave equation with respect to some suitably defined inner product (,). j refers to the k_j mode. A canonical (Heisenberg) vacuum state is as usual defined by

$$a^{\mathrm{in}}_{j\lambda}|0;\mathrm{in}\rangle=0. \tag{12}$$

Similarly, in the infinite future we have

$$A^{\mu\,\mathrm{out}}=\sum_{j\lambda}\ (a^{\mathrm{out}}_{j\lambda}A^{\mu\,\mathrm{out}}_{j\lambda}+a^{\mathrm{out}\dagger}_{j\lambda}A^{\mu\,\mathrm{out}*}_{j\lambda}), \tag{13}$$

which defines a vacuum state by

$$a^{\mathrm{out}}_{j\lambda}|0;\mathrm{out}\rangle=0. \tag{14}$$

The descriptions of the electromagnetic field in terms of the in and out states are not in general physically equivalent, since the products $\beta_{j'\lambda';j\lambda}\equiv(A^{\mu\,\mathrm{in}}_{j'\lambda'},A^{\mu\,\mathrm{out}*}_{j\lambda})$ do not vanish in general.

Let us now impose the generalized Lorentz-gauge condition on the positive frequency part of the operators above, i.e., consider the general condition

$$(\partial_\nu A^{\nu(+)}+\epsilon\partial_t\phi^{(+)})|\Psi\rangle=0, \quad \nu\in\{1,2,3\}, \tag{15}$$

where $|\Psi\rangle$ is an arbitrary quantum state. $(+)$ indicates the positive frequency part of the operator. This condition reduces to the usual Gupta-Bleuer condition when $\epsilon=1$. In this true vacuum case the Gupta-Bleuer condition implies the relation

$$(a_{j3}^{\rm in}-a_{j0}^{\rm in})|\Psi\rangle=0,\forall j. \qquad (16)$$

Let us follow the arguments in [3], and assume that the system starts off in the true vacuum situation (i.e., $\epsilon_{\rm initial}\equiv 1$), and that the refractive index is turned on after some time. At $t\rightarrow+\infty$ the Gupta-Bleuer condition can naturally be written in terms of the out operators

$$(a_{i3}^{\rm out}-\sqrt{\epsilon_{\rm final}}a_{i0}^{\rm out})|\Psi\rangle=0,\forall i. \qquad (17)$$

The state vector is unchanged, since it is time independent in the Heisenberg picture. The operator condition in this last expression can be rewritten in terms of the operators appropriate for the true vacuum situation. This results in the expression

$$\left(\sum_j \alpha_{ij}(a_{j3}^{\rm in}-\sqrt{\epsilon_{\rm final}}a_{j0}^{\rm in})+\sum_j \beta_{ij}^{*}(a_{j3}^{\rm in}-\sqrt{\epsilon_{\rm final}}a_{j0}^{\rm in})^{\dagger}\right)|\Psi\rangle =0,\forall i. \qquad (18)$$

α_{ij} and β_{ij} are the Bogolubov coefficients that relate the in and out states (see [6], e.g.). The condition in Eq. (16) can be used to eliminate one of the in operators. We can then in particular reexpress Eq. (18) as

$$(1-\sqrt{\epsilon_{\rm final}})\sum_j (\alpha_{ij}a_{j(0,3)}^{\rm in}+\beta_{ij}^{*}a_{j(0,3)}^{\rm in\dagger})|\Psi\rangle=0,\forall i. \qquad (19)$$

(0,3) indicates that this condition holds for the scalar and longitudinal components separately. This condition can be further simplified, by appealing to the definitions of the Bogolubov coefficients, to read

$$(1-\sqrt{\epsilon_{\rm final}})a_{i(0,3)}^{\rm out}|\Psi\rangle=0,\forall i. \qquad (20)$$

Hence, consistency requires that we quantize the physical, i.e., the transversal, degrees of freedom in the out region in the usual way, but put the scalar and the longitudinal components *identically to zero,* as was indicated in Eq. (9). The physical content of the resulting quantum theory is of course the one we would expect. However, what is remarkable, and very unexpected, is that we arrived at our conclusions, Eqs. (9) and (10), without the need to appeal to gauge invariance, or the equations of motion, at $t\rightarrow+\infty$, as in the standard vacuum theory. It is unclear to us what the deeper significance of this property is. We emphasize that this feature is a general one, since the end result is *independent* of the details of how the dielectric constant ϵ changes with time.

We also note another feature which deserves to be mentioned. The $|\Psi\rangle$ state is constrained by two relations which involve in operators, namely the constraint in Eq. (16) and the one in Eq. (19). Since the $a_{j(0,3)}^{\rm in}$ operators are linearly independent, the condition in Eq. (19) must hold for each term in the sum separately, i.e.,

$$(\alpha_{ij}a_{j(0,3)}^{\rm in}+\beta_{ij}^{*}a_{j(0,3)}^{\rm in\dagger})|\Psi\rangle=0, \quad \forall i,j. \qquad (21)$$

It follows from these relations that the unphysical sector of $|\Psi\rangle$, $|\Psi\rangle^{\rm unphys}$ can be represented as a coherent state, i.e.,

$$|\Psi\rangle^{\rm unphys}=Ne^{\sum_{ij}\Gamma_{ij}(a_{j3}^{\rm in\dagger}a_{j3}^{\rm in\dagger}-a_{j0}^{\rm in\dagger}a_{j0}^{\rm in\dagger})}|0;{\rm in}\rangle. \qquad (22)$$

N is a nonvanishing normalization constant, and the Γ_{ij} coefficients are defined by

$$\Gamma_{ij}=-\frac{\beta_{ij}^{*}}{\alpha_{ij}}. \qquad (23)$$

When applied to this coherent state the condition in Eq. (16) implies

$$(a_{j3}^{\rm in\dagger}-a_{j0}^{\rm in\dagger})|\Psi\rangle^{\rm unphys}=0, \quad \forall j. \qquad (24)$$

What all this amounts to is that the initial configuration of scalar and longitudinal excitations must have a particular form. This form is dictated by the behavior of the dielectric constant in the future. We emphasize that this point is important since there was no guarantee *a priori* that the initial photon state does not contain physical scalar and longitudinal excitations as they are defined in the future. However, the model considered is consistent, at least at this level, since we can always choose the unphysical sector to have the form in Eq. (22). Let us now turn to the question of the nature of the emission spectrum from free and bound electrons which live in a medium with a time-dependent refractive index.

As mentioned above, in [3] the dielectric constant was assumed to be a position-independent parameter, enabling one to obtain a rough order of magnitude of the energy emission from the medium. In order to actually understand what triggers the emission of photons, one must look at the microscopic dynamics more closely. There are two different physical mechanisms that are very natural to consider: first, the molecules in the bubble may be excited, eventually ionized, by photonic interaction. Second, the molecules may experience this effect because of fluid dynamical forces, induced by shock waves. As for the latter possibility, the fluid dynamical theory of sonoluminescence given by Kwak and coworkers [13], based upon the theory of dense gas in the bubble as given, in particular by Wu and Roberts [14], is quite impressive. Now, it is also an experimental fact that some amount of noble gases is required in the bubble to make the sonoluminescent process work. What is the physical reason for this? The explanation is as yet not completely clear, but is most probably related to the fact that the spherical symmetry of the noble gas atoms prevent the excitation energy from becoming distributed over rotational or vibrational modes. The rotational symmetry thus helps to make the emitted frequencies high.

From these physical considerations we adopt henceforth the following model: there exists a thin gas of free electrons in the bubble; we do not specify whether it is the result of photoionization, shock wave effects, or a combination of these. Mathematically, we take the initial quantum state at sufficiently early times to be

$$|i\rangle\equiv|1;{\rm in}\rangle\otimes|0;{\rm in}\rangle, \qquad (25)$$

where we have suppressed all quantum numbers. The first vector in the tensor product describes the number of initial electrons, while the second vector designates the number of initial photons. We write the final state in the same vain as

$$|f\rangle \equiv |1';\mathrm{out}\rangle \otimes |n;\mathrm{out}\rangle, \tag{26}$$

where we have put a prime on the fermionic state in order to distinguish it further from the initial fermionic state. The interaction term between the electron field and the electromagnetic field in the bulk is

$$\mathcal{H}_{\mathrm{I}} = A_\mu j^\mu = e A_\mu \overline{\psi} \gamma^\mu \psi, \tag{27}$$

where e is the fundamental electric charge. ψ denotes the electron-positron field, $\overline{\psi}$ its conjugate, and γ^μ represent the usual gamma matrices. This interaction is the only one we will consider. Hence, we neglect any effects that are associated with having to deal with a finite volume, or effects that appear due to the motion of the surface of the confining volume. However, we will briefly return to these issues at the end of the paper.

The asymptotic properties of the dielectric constant are characterized by Eqs. (7) and (8). The incoming electromagnetic vacuum state, as it is canonically defined at $t \to -\infty$, is in general related to the outgoing electromagnetic vacuum state, as it is defined at $t \to +\infty$ by [see Eq. (7.9) in [15] for the case when the α and β matrices are diagonal. It is straightforward to generalize that expression to the one in Eq. (28)]

$$(\phi_0)^{-1} |0;\mathrm{in}\rangle = e^{[1/2 \sum_{ij\lambda} \gamma^*_{ij\lambda} a^{\mathrm{out}\dagger}_{j\lambda} a^{\mathrm{out}\dagger}_{-j\lambda}]} |0;\mathrm{out}\rangle. \tag{28}$$

ϕ_0 is in general an arbitrary constant, which we will set equal to unity, * denotes complex conjugation, and $\gamma_{ij} \equiv (\beta_{ij}/\alpha_{ij})^*$. This relation follows due to the completeness of the Fock spaces. From the expression for $|0;\mathrm{in}\rangle$ in Eq. (28) we see in particular that any energy emission (in the form of photons as they are defined in the future) from $|0;\mathrm{in}\rangle$ in the far future must involve correlated pairs of photons. In dielectric media it is only the electromagnetic fluctuations that are changed when one compares with the pure vacuum case, i.e., when considered as free fields it is only the second quantization of the electromagnetic field which is modified when $\epsilon > 1$, while the second quantization of the electron-positron field is identical to the one in the true vacuum. Hence, the part of the Fock space that is associated with the incoming electron in Eq. (25) is not altered during the change in the refractive properties of the medium. It is thus appropriate to change the notation in the definition of $|f\rangle$ into

$$|f\rangle = |1';\mathrm{in}\rangle \otimes |n;\mathrm{out}\rangle. \tag{29}$$

Photon emission from a free electron at the tree level in the out region is described by the S-matrix element

$$S_{fi} \equiv \int \langle f|\mathcal{H}_{\mathrm{I}}|i\rangle, \tag{30}$$

where $\int$ indicates integration over all space time. By keeping just the first nontrivial term in Eq. (28) the S-matrix element above can be expanded as

$$\begin{aligned} S_{fi} &= \int \langle f|\mathcal{H}_{\mathrm{I}}|1';\mathrm{in}\rangle \otimes |0;\mathrm{out}\rangle \\ &\quad + \tfrac{1}{2} \int \langle f|\mathcal{H}_{\mathrm{I}} \left(\sum_{ij\lambda} \gamma^*_{ij} a^\dagger_{j\lambda} a^\dagger_{-j\lambda} \right) |1';\mathrm{in}\rangle \otimes |0;\mathrm{out}\rangle + \cdots \\ &\equiv S_{fi(1)} + S_{fi(2)} + \cdots . \end{aligned} \tag{31}$$

Since the electron-positron field is insensitive to a changing refractive index we will neglect this part in the following in order to simplify our formulas. At $t \to -\infty$ the $\{A^\mu\}$ potentials have the general form in Eq. (11). These potentials are propagated forward in time by the equations of motion which take an explicitly time-dependent refractive index into account (these equations are reproduced in [3]). In order to compute the S-matrix we utilize the relation

$$\begin{aligned} A^\mu &= \sum_{j\lambda} (a^{\mathrm{in}}_{j\lambda} A^{\mu \mathrm{in}}_{j\lambda} + a^{\mathrm{in}\dagger}_{j\lambda} A^{\mu \mathrm{in}*}_{j\lambda}) \\ &= \sum_{ij\lambda} \{ [\alpha_{ij} A^{\mu \mathrm{in}}_{j\lambda} + (\beta_{ij} A^{\mu \mathrm{in}}_{j\lambda})^*] a^{\mathrm{out}}_{j\lambda} \\ &\quad + [\beta_{ij} A^{\mu \mathrm{in}}_{j\lambda} + (\alpha_{ij} A^{\mu \mathrm{in}}_{j\lambda})^*] a^{\mathrm{out}\dagger}_{j\lambda} \}, \end{aligned} \tag{32}$$

where in the last two lines we have expressed the in operators in terms of the out operators. The $A^{\mu \mathrm{in}}_{j\lambda}$ modes have been calculated for a particular case in [3] in terms of hypergeometric functions. The relevant part of the S matrix that describes emission of a single photon from an electron is then given by

$$\begin{aligned} S_{fi(1)} = \int \Bigg(&\langle \mathrm{out};1| \sum_{ij\lambda} [\beta_{ij} A^{\mu \mathrm{in}}_{j\lambda} + (\alpha_{ij} A^{\mu \mathrm{in}}_{j\lambda})^*] a^{\mathrm{out}\dagger}_{j\lambda} |0;\mathrm{out}\rangle \\ &\otimes \mathrm{fermions}_\mu \Bigg). \end{aligned} \tag{33}$$

Such emission at the tree level is in the literature called transition radiation and is a well known phenomenon, although our quantum field theoretical treatment of it seems to be among the first ones. After $S_{fi(1)}$ has been computed it is straightforward to compute $S_{fi(2)}$ which describes the emission of three photons of which two are back to back. When still further terms in Eq. (28) are taken into account we find that an electron may emit any odd number of photons.

Sonoluminescent bubbles may consist entirely of noble-gas atoms [16]. Such atoms have ionization energies of the order of 15 eV. As the observed radiation in experiments with sonoluminescent bubbles have a typical energy of the order of 3–4 eV, it can be argued (see [3], e.g.) that the presence of free charges in a sonoluminescent bubble is an unlikely hypothesis. However, in practice, with a mixture of gases in the bubble, the complexity of the interactions (photonic or hydrodynamic) makes it conceivable that there exists a thin electron gas there nevertheless. And, as mentioned, this is at the core of our model.

It was argued in [17] that it is in principle possible to experimentally verify whether the vacuum picture of sonoluminescence is a viable one by measuring two-photon correlations. Our main aim in this paper is to investigate whether the inclusion of free or bound charges will induce further properties to the radiation which is emitted by such bubbles. Transition radiation (meaning the emission of a *single* photon from a charge) is a well known physical effect which will be present in *any* dielectric medium with a time-dependent refractive index as soon as free electric charges are present. On the assumption of the presence of a thin gas of electrons inside sonoluminescent bubbles it follows that transition radiation must also be taken into account in order to understand sonoluminescence. In this paper we have shown that a quantum treatment of transition radiation reveals that a free electron does not necessarily emit only a single photon, but it may in fact emit any odd number of photons. This finding opens up new directions for further experimental work. Clearly, since electrons can emit any odd number of photons it will be induced correlations between odd number of photons when one measures the *arrival times* of photons from sonoluminescent bubbles.

It may seem straightforward, and therefore tempting, to compute the efficiency of the production of the extra correlated photon pairs in transition radiation processes by putting the Bogolubov coefficients which were computed in [3] into our formulas. We want to emphasize that the inclusion of the accelerated cavity walls may be of *crucial importance* for such estimates. However, transition radiation in connection with accelerated layer surfaces appears to be a completely uncharted area in the literature. Hence, realistic estimates of the efficiency of the transition radiation mechanism to produce correlated pairs of photons must await future studies of this particular problem [18].

[1] B.P. Barber and S.J. Putterman, Phys. Rev. Lett. **69**, 3839 (1992).
[2] J. Schwinger, Proc. Natl. Acad. Sci. USA **90**, 958 (1993); **90**, 2105 (1993); **90**, 4505 (1993); **90**, 7285 (1993); **91**, 6473 (1994).
[3] S. Liberati, M. Visser, F. Belgiorno, and D.W. Sciama, Phys. Rev. D **61**, 85023 (2000).
[4] I. Brevik, Phys. Rep. **52**, 133 (1979).
[5] S. Liberati, M. Visser, F. Belgiorno, and D.W. Sciama, Phys. Rev. D **61**, 85024 (2000).
[6] N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1986).
[7] L. Parker, Phys. Rev. Lett. **59**, 1369 (1987); I. Brevik and T. Toverud, Phys. Rev. D **51**, 691 (1995); I. Brevik and B. Jensen, *ibid.* **59**, 045010 (1999).
[8] L. Motyka and M. Sadzikowski, e-print physics/9912013.
[9] V.L. Ginzburg and V.N. Tsytovich, Phys. Rep. **49**, 1 (1979).
[10] V.L. Ginzburg, in *Applications of Electrodynamics in Theoretical Physics and Astrophysics* (Gordon and Breach, New York, 1987).
[11] I. Brevik, V.N. Marachevsky, and K.A. Milton, Phys. Rev. Lett. **82**, 3948 (1999).
[12] C.E. Carlson, C. Molina-París, J. Pérez-Mercader, and M. Visser, Phys. Lett. B **395**, 76 (1997); Phys. Rev. D **56**, 1262 (1997); C. Molina-París and M. Visser, *ibid.* **56**, 6629 (1997).
[13] H. Kwak and J. Na, Phys. Rev. Lett. **77**, 4454 (1996); H. Kwak and H. Yang, J. Phys. Soc. Jpn. **64**, 1980 (1995).
[14] C.C. Wu and P.H. Roberts, Q. J. Mech. Appl. Math. **49**, 501 (1996).
[15] S.A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time*, London Mathematics Society Study Texts Vol. 17 (Cambridge University Press, Cambridge, England, 1989).
[16] D. Lohse and S. Hilgenfeldt, J. Chem. Phys. **107**, 6986 (1997); T.J. Matula and L.A. Crum, Phys. Rev. Lett. **80**, 865 (1998); J.A. Ketterling and R.E. Apfel, *ibid.* **81**, 4991 (1998).
[17] F. Belgiorno, S. Liberati, M. Visser, and D.W. Sciama, e-print quant-ph/9904018.
[18] B. Jensen and I. Brevik (unpublished).